2023

PHYSICS — HONOURS

Paper : DSCC-1

(Basic Physics-I)

Full Marks : 75

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.* 1** and ***any five*** questions, taking at least ***one*** question from ***each Group***.

1. Answer ***any five*** questions : 3×5

(a) Sketch schematically the function $f(x) = |x|^{\beta}$ (where β is a parameter) in a single plot for three different values of $\beta = ½, 1, 2$, in the range $-1 \le x \le +1$. For which of the above values of β is the function differentiable at the origin?

(b) If $\vec{a} \times \vec{g} = \vec{b}$ and $\vec{a} \cdot \vec{g} = \Phi$, express $\vec{g}$ in terms of $\vec{a}, \vec{b}, \Phi$ and $|\vec{a}|$.

(c) Expand $\dfrac{1}{x-2}$ in a Taylor series about the point $x = 1$.

(d) Solve the differential equation $\dfrac{dy}{dx} + 2y = x^2$.

(e) A particle moves in 2-dimensions on the ellipse : $x^2 + 4y^2 = 1$. At a particular instant, it is at the point $(x, y) = (2, 1)$ and the x-component of its velocity is 2 (in suitable units). Find the y-component of its velocity at that instant.

(f) Show that the centre of mass of a system of particles is unique.

(g) Show that the areal velocity of a particle moving in x-y plane under a central force is constant.

(h) A horizontally placed hollow tube has a cross-sectional area A at the beginning of the tube that gradually tapers off to $\frac{A}{2}$ at the end. An incompressible, ideal fluid of density ρ enters the tube with a velocity v at the beginning of the tube. What is the difference in pressure at the two ends of the tube?

Group - A

2. (a) Evaluate $\lim\limits_{x \to 0} \dfrac{1 - \cos^n x}{x^2}$.

(b) Consider a function $f(x) = Ae^{-\lambda x}$ with the constraint $\int_{-\infty}^{\infty} f^2(x)dx = 1$. x has the dimension of length. Using dimensional analysis, find dimension of A in terms of λ.

(c) Examine whether the differential equation $x(x^2 + 2y^2)\, dx + y\,(2x^2 + y^2)\, dy = 0$ is exact. Solve the equation, if exact.

(d) Given that $x''(t) - 4x'(t) + 4x(t) = 0$ with $x(0) = 1$ and $x'(0) = 0$, find out $x\left(-\frac{1}{2}\right)$, where $'$ indicates derivative with respect to t. 2+3+(1+2)+4

3. (a) Using the summation convention and Levi-Civita symbol, show that $\vec{A}\cdot\left(\vec{A}\times\vec{B}\right) = 0$.

(b) If $\vec{A}$ is an irrotational vector field, show that $\vec{A}\times\vec{r}$ is solenoidal.

(c) Given that $\Phi\,(x, y, z) = xy + \sin z$, find $\vec{\nabla}\Phi$ at $\left(1, 2, \frac{1}{2}\pi\right)$. How fast is Φ increasing in the direction of $4\hat{i} + 3\hat{j}$ at $\left(1, 2, \frac{1}{2}\pi\right)$?

(d) Prove that $\oiint_S d\vec{S} = 0$ for any closed surface S. 3+3+(2+1)+3

4. (a) Calculate $\iint_S \vec{\nabla}\times\vec{A}\cdot d\vec{S}$, where $\vec{A} = -y\hat{i} + x\hat{j}$ and S is the open surface area of hemisphere $x^2 + y^2 + z^2 = a^2,\ z \geq 0$.

(b) Find the expression of $\frac{\partial}{\partial x}$ in terms of plane polar coordinates.

(c) Suppose a particle is moving along a trajectory given by $r = ct$, $\theta = \Omega\, t$, where c and Ω are positive constants. Find the velocity and acceleration vectors of the particle at time t. 4+4+(2+2)

Group - B

5. (a) A particle of mass m is falling under gravity in presence of a drag force $-\gamma v$, where v is the speed and $\gamma(>0)$ is a constant.

(i) Set up the equation of motion of the particle.

(ii) Find the speed $v(t)$ at a later time t assuming $v(t = 0) = 0$.

(iii) Show that $v(t\rightarrow\infty) = \frac{mg}{\gamma}$.

(iv) Plot $v(t)$ against t.

(v) From the expression of $v(t)$ obtained, recover the familiar expression of speed in the drag-free case.

(b) A particle is moving on a circular path with a constant speed u. Find the magnitude of the change in its velocity after it has swept an angle θ. (1+3+1+2+2)+3

6. (a) A particle moves in 1-dimension in a potential $V(x)$ having a local minima at x_0. Show that for small displacements around x_0, the particle behaves like a harmonic oscillator.

(b) A particle of mass m is moving in the region $x > 0$ under the influence of the potential $U(x) = U_0\left(\frac{\alpha}{x} + \frac{x}{\alpha}\right)$, where U_0 and α are positive constants. (i) Find the force acting on the particle. (ii) Find the positions of equilibrium and identify the stable one. (iii) Find the frequency of small oscillation about this point.

(c) Show that for a particle moving in a conservative force field $\vec{F}$, the integral $\int_1^2 \vec{F}\cdot d\vec{r} = T_2 - T_1$, where T is the kinetic energy of the particle. 4+(1+2+1)+4

7. (a) The position vector of a moving particle at any instant of time t is given by $\vec{r} = \left(2+3t^2\right)\hat{i} + 5t^2\hat{j} + t\hat{k}$. Find the force $\vec{F}$, torque $\vec{N}$ and angular momentum $\vec{L}$ of the particle about the origin. Hence, verify that $\vec{N} = \frac{d\vec{L}}{dt}$.

(b) For a system of particles, show that the angular momentum about a point is equal to the angular momentum of a single particle of total mass $M\left(=\sum_i m_i\right)$ situated at the centre of mass together with the angular momentum of the system of particles about the centre of mass.

(1+2+2+1)+6

8. (a) A particle is thrown from the Earth's surface with speed $v = \sqrt{\frac{3GM}{2R}}$, where M and R are mass and radius of the Earth respectively. What will be the nature of the orbit of the particle?

(b) Find the central force for which the orbit is given by $r = ke^{a\theta}$, where a and k are constants.

(c) A planet of mass m moves around the Sun of mass M. The nearest and the farthest distance of the planet from the Sun are a and b respectively. Find the magnitude of the angular momentum of the planet around the Sun in terms of m, M, a, b and G, where G is the gravitational constant.

(d) A ball moving with speed of 9 m/s strikes an identical stationary ball such that after the collision, the direction of each ball makes an angle 30° with the original line of motion. Find the speeds of two balls after collision. Is the kinetic energy conserved in this collision? 2+3+3+(2+2)

9. (a) Set up the equation of continuity expressing local conservation of mass, in the context of fluid motion.

(b) Explain Newtonian liquid and non-Newtonian liquid with one example in each case.

(c) (i) Establish Euler's equation for an ideal fluid moving in the presence of gravity.

(ii) Hence, deduce that in a fluid at rest, the pressure difference between two points inside the fluid, separated by a vertical height h is $h\rho g$, where ρ is the density of the fluid and g is acceleration due to gravity. 3+(2+1)+(4+2)