2025

MATHEMATICS — HONOURS

Paper : DSCC-2

(Basic Algebra)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meaning.

Group - A

1. Answer ***any two*** questions : 2½×2

   (a) Find the *modulus* and principal amplitude of the complex number $Z = 1 + i\tan\theta$, $\frac{\pi}{2} < \theta < \pi$.

   (b) For positive real numbers a, b, c, find the least value of $a^{-1} + b^{-1} + c^{-1}$ if $a + b + c = 2020$.

   (c) Find an equation whose roots are reciprocal of the roots of the equation $x^3 + 5x^2 - 8x + 10 = 0$.

2. Answer ***any four*** questions : 5×4

   (a) If $x = \log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$, where θ is real, then prove that $\theta = -i\,\mathrm{Log}\tan\left(\frac{\pi}{4} + i\frac{x}{2}\right)$.

   (b) If $\sin(\theta + i\varphi) = \tan\beta + i\sec\beta$, then prove that $\cos 2\theta \cosh 2\varphi = 3$.

   (c) If x, y and z are positive real numbers and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, then show that the least value of $x^2 + y^2 + z^2$ is 27.

   (d) If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, then find the equation whose roots are $\frac{\beta+\gamma}{\alpha^2}, \frac{\gamma+\alpha}{\beta^2}, \frac{\alpha+\beta}{\gamma^2}$.

   (e) Solve the equation, $x^4 + 4x^3 - 6x^2 + 20x + 8 = 0$ by Ferrari's method.

   (f) By Sturm method prove that the roots of the equation $x^3 - (a^2 + b^2 + c^2)x - 2abc = 0$ are all real.

Group - B

3. Answer ***any two*** questions : 2½×2

(a) If S be a finite set with n elements, then find the number of reflexive relations that can be defined on S.

(b) Find the remainder when $6(7)^{32} + 7(9)^{45}$ is divided by 4.

(c) For any prime p and for any two integers a, b if p/ab, then show that either p/a or p/b.

4. Answer ***any four*** questions :

(a) Let S be the set of all positive divisors of 36. On S define a relation ρ by $a\rho b$ if and only if $a \mid b$; for $a, b \in S$. Prove that (S, ρ) is a poset. Is it a linear ordered set? Justify your answer. 3+2

(b) (i) Show that an equivalence relation on a non-empty set induces a partition on the set.

(ii) A relation ρ on $\mathbb{Z}$ is defined by $a\rho b$ if and only if $ab \geq 0$. Is ρ an equivalence relation on $\mathbb{Z}$? Justify. 3+2

(c) (i) If $gcd\,(a, b) = 1$, then show that $gcd\,(a + b, a - b) = 1$ or 2.

(ii) Prove that the number of prime is infinite. 2+3

(d) Solve : $x \equiv 2\ (mod\ 3)$; $x \equiv 3\ (mod\ 5)$; $x \equiv 4\ (mod\ 7)$. 5

(e) If $n = p_1^{a_1}\ p_2^{a_2}\ ......p_r^{a_r}$ and $p_1, p_2, ...\ p_r$ are prime to each other, then prove that

$$\varphi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)...\left(1 - \frac{1}{p_r}\right),$$

where φ is Euler's phi function. 5

(f) Find $\sigma(900)$ and $\tau(900)$. 5

Group - C

5. Answer ***any two*** questions : 2½×2

(a) Express the vector (1, 1, 1) as a linear combination of the vectors (2, 1, 1) and (1, 2, 2) in $\mathbb{R}^3$.

(b) Identify the free variable of the system of equation

$$2x + y + 12z = 1$$
$$x + 2y + 9z = -1$$

(c) Find the value of k for which the system of equations

$$x + y + z = 0$$
$$y + 2z = 0$$
$$kx + z = 0$$

has more than one solution.

6. Answer ***any four*** questions : 5×4

(a) Determine the conditions for which the system of equations given below has (i) only one solution, (ii) no solution and (iii) many solutions.

$$x + 2y + z = 1$$
$$2x + y + 3z = b$$
$$x + ay + 3z = b + 1$$

(b) Find a row-reduced echelon form of the matrix

$$\begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}$$

and find its rank.

(c) If A is an $n \times n$ invertible matrix, then prove that A has n pivot positions.

(d) Solve the system of equations

$y + z = a,\ x + z = b,\ x + y = c,$

and use the solution to find the inverse of the matrix.

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

(e) Consider the vectors (1, 2), (2, 3), (3, 4), (4, 5) in $\mathbb{R}^2$. Let P be the set spanned by the vectors (1, 2), (2, 3) and Q be the set spanned by the vectors (3, 4), (4, 5). Examine whether $P = Q$.

(f) Find all real λ for which the rank of the matrix A is 2, when

$$A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 5 & 3 & \lambda \\ 1 & 1 & 6 & \lambda+1 \end{pmatrix}$$

Hence, solve the following system of equations for that value(s) of λ :

$$x + 2y + 3z = 1$$
$$2x + 5y + 3z = \lambda$$
$$x + y + 6z = \lambda + 1.$$
