2024

PHYSICS — HONOURS

Paper : DSCC-4

(Mathematical Physics - I)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any five*** questions from the rest.

1. Answer ***any five*** questions : 3×5

(a) Determine the condition under which the differential equation

$$C_1 \frac{\partial y}{\partial t} + C_2 \frac{\partial^2 y}{\partial t^2} + v(x, t)\, y(x, t) = 0,$$ can be solved using method of separation of variables.

(b) A dice is thrown 8 times. What is the probability that 4 will appear at least 6 times?

(c) Define the step function. Hence show that, the derivative of the step function can be written as a Dirac delta function.

(d) Show that $\sin nx$ and $\cos mx$ are always orthogonal in the range $[0, 2\pi]$ for $m \neq n$ as well as $m = n$.

(e) Define odd function. Show that expansion of any odd function in the range $-\pi \leq x \leq \pi$ in Fourier series consists of only sine terms.

(f) Write down one for each elliptic type, hyperbolic type and parabolic type partial differential equation.

(g) Show that average value $\bar{x}$ is zero for Gaussian distribution $Ae^{-\alpha x^2}$.

(h) Explain with example :

(i) Truncation error,

(ii) Rounding error, and

(iii) Propagation error.

2. (a) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent, but $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent.

(b) Test the convergence of the series

$1^2 + 2^2x + 3^2x^2 + 4^2x^3 + ...$ for $x < 1$, $x > 1$ and $x = 1$.

(c) Let a Geometric series is given by $S_n = 1 + r + r^2 + ... + r^{n-1}$.

Show that the series oscillates only when $r \leq -1$. (3+2)+3+4

3. (a) What are odd and even functions? Explain.

(b) Find the Fourier series expansion for the function

$f(x) = |x|$ in $-\pi \leq x \leq \pi$. Hence show that $1 + \frac{1}{3^2} + \frac{1}{5^2} ...... = \frac{\pi^2}{8}$.

Is this function differentiable everywhere?

(c) If a real function $f(x)$ be expanded into a Complex Fourier Series as $f(x) = \sum_{-\infty}^{+\infty} C_n e^{inx}$,

then show that $C_{-n} = C_n^*$. 2+(4+2+1)+3

4. (a) Explain whether $\tan x$ can be expressed in Fourier Series or not in $-\pi \leq x \leq \pi$.

(b) Let $F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ikx} f(x) dx$ be the Fourier transform of $f(x)$.

Show that $\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(k)|^2 dk$.

(c) Consider a function $f(x) = 1$ in the range $-a < x < a$ and zero elsewhere. Find Fourier transformed function of $f(x)$.

(d) If $F(k)$ is the Fourier transform of a function $f(x)$, then prove that the Fourier transform of $\frac{df(x)}{dx}$

is $-ikF(k)$. Assume that for $x \to \pm\infty$, $f(x) \to 0$. 2+3+4+3

5. (a) Solve the equation $\frac{\partial^2 U}{\partial x^2} = \frac{1}{2}\frac{\partial U}{\partial t}$ by method of separation of variables with boundary conditions

U (0, t) = 0, U (3, t) = 0 and U (x, 0) = 5 sin $4\pi x$, where $0 < x < 3$, $t > 0$.

(b) Solve the partial differential equation in two dimension

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

using method of separation of variables. Using the boundary conditions

(i) $\phi(0, y) = 0$, (ii) $\phi(a, y) = 0$, (iii) $\phi(x, \infty) = 0$, (iv) $\phi(x, 0) = V_0$

show that solution leads to the general form

$$\phi(x, y) = \sum_{n=1}^{\infty} A_n \sin(n\pi x/a) e^{-n\pi y/a}$$ and hence find A_n. 6+6

6. (a) The normal distribution is defined as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \text{ where } -\infty < x < \infty.$$

Show that the mean = μ and standard deviation = σ.

(b) Plot $f(x)$ vs x for $\mu = -1, +1$ with same σ and also plot $f(x)$ vs x keeping $\mu = 0$ with $\sigma = \frac{1}{2}, 1$.

(c) Show that under certain conditions Binomial distribution can be converted into Poisson's distribution.

(d) A random variable x has the probability density function $f(x) = \begin{cases} Cx; & 0 \le x \le 2 \\ 0; & \text{otherwise} \end{cases}$

Find (i) the constant C

(ii) the probability when $x > 1$. (2+2)+(1+1)+3+(1+2)

7. (a) Evaluate the integral $\int_{-0.5}^{0.5} \left(x^2 - 5x + 6\right) \delta(x-1) dx$

(b) Prove $\delta(ax) = \frac{1}{|a|}\delta(x)$ when $a \neq 0$

(c) Show that $\delta\left(x^2 - a^2\right) = \frac{1}{2a}\left(\delta(x-a) + \delta(x+a)\right)$

(d) Find $\int_0^{2\pi} \cos x \, \delta\left(x^2 - \pi^2\right) dx$

(e) Show that $\nabla^2\left(\frac{1}{r}\right) = -4\pi\delta^3(\vec{r})$. 1+3+3+1+4

8. (a) Write the integral $\int_0^1 \frac{x^3}{\sqrt{1-x}}\,dx$ in the form of a Beta function and hence evaluate it.

(b) Show that $\Gamma(n+1) = n\,\Gamma(n)$.

(c) Taking step size $h = 1.5$ evaluate the integral

$$\int_0^3 \left(x^2 - 3x + 4\right)dx$$

using (i) Trapezoidal rule and (ii) Simpson $\frac{1}{3}$ rule. 3+2+(3+4)

9. (a) Use Gauss-Seidel method to solve the equations $x + y + z = 9$; $2x - 3y + 4z = 13$; $3x + 4y + 5z = 40$. Start from the approximate solution (3, 3, 3) and use three iterations.

(b) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$

with initial condition $y = 1$ at $x = 0$; find y for $x = 0.1$ using Euler's method for the step size 0.02.
6+6
