BAGNAN COLLEGE ESTD 1958 Bagnan, Howrah

B(3rd Sm.)-Mathematics-H/DSCC-4/CCF

2024

MATHEMATICS — HONOURS

Paper : DSCC-4

(Ordinary Differential Equations - I and Group Theory - I)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer ***any nine*** questions.

1. Find the differential equation of the circles having centres on the line $y + x = 0$ and passing through the origin. 5

2. If the integrating factor of $(x^7y^2 + 3y)dx + (3x^8y - x)dy = 0$ is $x^m y^n$, then find the values of m and n. Further, for these values of m and n solve the equation. 5

3. Solve the following differential equation : 5

$$\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$$

4. Reduce the equation $x^3p^2 + x^2yp + a^3 = 0$ to Clairaut's fo rm by the substitution $y = u$ and $x = \frac{1}{v}$ and obtain the complete primitive. 5

5. Find the general solution of the differential equation $y(4x + y)dx - 2(x^2 - y)dy = 0$. 5

6. Use D-operator to solve : 5

$$\frac{d^2y}{dx^2} - y = x\sin x + \left(1+x^2\right)e^x.$$

7. Solve using the method of undetermined coefficient

$$(D^2 - 2D + 3)y = x^2 + \sin x \quad \left(D \equiv \frac{d}{dx}\right).$$ 5

BAGNAN COLLEGE ESTD 1958 Bagnan, Howrah

Please Turn Over

BAGNAN COLLEGE ESTD 1958 Bagnan, Howrah

8. Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2} + a^2y = \tan ax.$$ 5

9. Transform $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - y = 3x^3\cos(\log x)$ into a linear equation with constant coefficient by a suitable substitution and hence solve it. 2+3

10. Verify that the equation $\left(2x^2 + 3x\right)\frac{d^2y}{dx^2} + (6x+3)\frac{dy}{dx} + 2y = (x+1)e^x$ is exact and hence solve it. 2+3

11. Solve the following by changing the independent variables : 5

$$(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$$

12. Factorise the differential expression and obtain the complete primitive : 2+3

$$(x+2)y_2 - (2x+5)y_1 + 2y = (x+1)e^x$$

13. Solve the system of simultaneous equation : 5

$$\left.\begin{aligned} Dx - 7x + y = 0 \\ Dy - 2x - 5y = 0 \end{aligned}\right\},\ \text{where}\left(D \equiv \frac{d}{dt}\right)$$

14. Show that the equation of the curve, whose slope at any point (x, y) is equal to $xy(x^2y^2 - 1)$ and which passes through the point $(0, 1)$ is $x^2y^2 = 1 - y^2$. 5

Group - B

(Group Theory)

Answer ***any six*** questions.

15. (i) Prove that the set of all odd integers forms a commutative group with respect to '*' defined by $a * b = a + b - 1 \vee a, b \in D$.

(ii) In a group $(G, *)$ each element is its own inverse. Prove that the group is commutative. 3+2

16. Let H and K be subgroups of a group G. Then prove that HK is a subgroup of G if and only if $HK = KH$. 5

17. Let (G, o) be a group of H be a non-empty finite subset of G. Then prove that (H, o) is a subgroup of (G, o) if and only if $a \in H, b \in H \Rightarrow a \circ b \in H$. 5

18. What is centre of a group? Prove that it forms a subgroup of the group. 1+4

19. Prove that a non-commutative group of order 10 must have a subgroup of order 5. 5

20. (i) Let $G = \langle a \rangle$ and $O(G) = 30$, then find the order of the cyclic subgroup generated by a^{18}.

(ii) Let G be an abelian group of order 6, containing an element of order 3. Prove that G is a cyclic group. 2+3

21. (i) Show that every group of prime order is cyclic.

(ii) Give an example of a non-cyclic group in which all proper subgroups are cyclic subgroups. 3+2

22. (i) Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 8 & 5 & 6 & 4 & 7 & 1 \end{pmatrix}$ on S_8 as a product of transpositions and find the order.

(ii) Find the largest order of an element in the group S_6. (2+1)+2

23. (i) Let (G, o) be a group and $a, b \in G$. If $o(a) = 3$ and $a_o b_o a^{-1} = b^2$, find $o(b)$ if $b \neq e$.

(ii) Find all the distinct cosets of the subgroup $H = \{e, (1, 2, 3), (1, 3, 2)\}$ in the subgroup S_3. 4+1

BAGNAN COLLEGE ESTD 1958 Bagnan, Howrah