2025

PHYSICS — HONOURS

Paper : DSCC-8

(Classical Mechanics and Special Theory of Relativity)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any five*** questions from the rest.

1. Answer ***any five*** questions :

   (a) Show that if the earth revolves about 17 times faster than as it does now, the weight of a body on the equator would be zero (equatorial radius of earth is 6400 km). 3

   (b) What do you mean by pure translation and pure rotation of a rigid body? Illustrate with corresponding diagrams. 3

   (c) Write down the expression of the inertia tensor of a system of N particles each with different mass m_i, $i = 1,...,N$. Show that the tensor is symmetric. 1+2

   (d) What is a functional? Illustrate with one example. 2+1

   (e) Using extremization of action as a basic principle in mechanics, show that the Lagrangian $L(q,\dot{q})$ and $L(q,\dot{q})+\dfrac{dF}{dt}$ lead to the same equation of motion where F is a function of the generalized coordinate q only. Here, $\dot{q}$ is the generalized velocity. 3

   (f) The Lagrangian of a system is given by

   $$L=\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2.$$

   Find the conjugate momentum and the Hamiltonian. 1+2

   (g) Draw a schematic diagram of a Michelson-Morley interferometer. Explain the significance of the experiment. 2+1

   (h) Explain whether there is any violation of the principle of causality in relativity for two events separated by 'space-like' interval. 3

2. (a) A car in linear motion is accelerating with respect to ground with acceleration $\vec{a}$. Obtain the expression for the fictitious force acting on a particle of mass m in the car.

   (b) A pendulum is hung from the roof of the above car. What is the equilibrium angle that the string of the pendulum makes with the vertical and what is the tension in the string?

(c) Show that the time derivative of a vector $\vec{A}(t)$ in a fixed and in a rotating coordinate system are related as

$$\left(\frac{d\vec{A}}{dt}\right)_{Fixed} = \left(\frac{d\vec{A}}{dt}\right)_{Rot} + \left(\vec{\omega} \times \vec{A}\right),$$

where $\vec{\omega}$ is the angular velocity of the rotating frame.

(d) Explain the effect of the Coriolis force on the formation of cyclones in the northern hemisphere.

2+3+4+3

3. (a) The moments and products of inertia of a rigid body with respect to xyz coordinate system are I_{xx}, I_{yy}, I_{zz} and I_{xy}, I_{yz}, I_{zx}, respectively. Prove that the moment of inertia of the body about an axis making angles α, β, γ with x, y and z axes respectively is given by

$$I = I_{xx}\cos^2\alpha + I_{yy}\cos^2\beta + I_{zz}\cos^2\gamma + 2I_{xy}\cos\alpha\cos\beta + 2I_{yz}\cos\beta\cos\gamma + 2I_{zx}\cos\gamma\cos\alpha\,.$$

(b) For a homogeneous cube of mass m and side a (i) set up the principal axes with origin at the centre of mass of the cube. (ii) Calculate the principal moments of inertia I_1, I_2 and I_3.

(c) For rotational motion of a rigid body, derive an expression for kinetic energy in terms of moment of inertia and angular velocity.

4+(2+2)+4

4. (a) Show that a rigid body has six degrees of freedom in three dimensions.

(b) State and prove the parallel axes theorem for the moment of inertia tensor.

(c) Write down Euler's equation of motion for rotation of a rigid body. Express it using the principal axes of inertia.

(d) A rigid body with two equal principal moments of inertia ($I_1 = I_2$) is rotating about its center of mass with angular velocity $\vec{\Omega}$. Using Euler's equations of motion, show that the component of $\vec{\Omega}$ along the direction of the third moment of inertia remains constant in time.

2+(1+3)+(1+2)+3

5. (a) Using the variational principle, find the curve which represents the shortest distance between two points on a flat plane.

(b) A particle is constrained to move along the surface of a sphere. Identify a set of generalized coordinates. State the conditions that must be satisfied for a set of variables to represent generalized coordinates.

(c) Set up the Lagrangian for a simple pendulum and obtain an equation describing its motion.

4+(1+2)+(2+3)

6. (a) A particle of mass m is moving under a central force field whose potential is given by $V(r)$. Write down its Lagrangian in plane polar coordinate system (r, θ). Identify the ignorable coordinate and hence show that the angular momentum of the particle is a constant of motion.

(b) Prove that if the Lagrangian of a system is not explicitly dependent on time, then the Hamiltonian of that system is conserved.

(c) Prove that if the Lagrangian of an unconstrained system is invariant under continuous translation then its total linear momentum is conserved.

(d) The point of support (massless) of a simple pendulum (having mass m and weightless string of length l) is moving along a horizontal line according to the equation $x = a\cos\omega t$. Write the Lagrangian of the system.

4+2+4+2

7. (a) What is a Legendre transformation?

(b) Starting from the Lagrangian of a system with n degrees of freedom, obtain the Hamiltonian using Legendre transformation.

(c) Find the conditions on the real parameters α, β, γ and δ, such that

$$\dot{q} = \alpha q + \beta p$$
$$\dot{p} = \gamma q + \delta p$$

are Hamilton's equation of motion for some $H(p, q)$. Hence obtain $\dot{H}(p,q)$.

(d) If the kinetic energy $T = \frac{1}{2}m\dot{r}^2$ and potential energy $V = \frac{1}{r}\left(1 + \frac{\dot{r}^2}{c^2}\right)$, find the Hamiltonian H. Determine whether $H = T + V$.

2+3+4+3

8. (a) State the postulates of special theory of relativity.

(b) Define the space-time interval between two events in three spatial dimensions. Use the Lorentz transformation to show that the interval between two events remains the same in all inertial frames.

(c) Use the invariance of the interval between two events to derive the expression for time dilation.

(d) A pion is created at 200 km above the sea level. It descends vertically at a speed of 0.99 c and decays, in its proper frame, 2.5×10^{-8} sec after its creation. At what altitude above the sea level is it observed to decay?

2+(1+3)+3+3

9. (a) Use the relativistic velocity addition rule to show that the magnitude of sum of two velocities, each having magnitude less than the speed of light, can not exceed the speed of light.

(b) A stationary body explodes into two fragments each of mass 1 kg that move apart at a speed of 0.6 c relative to the original body. Find the mass of the original body.

(c) Write down the Lorentz transformation in Minkowski space and hence show that in Minkowski space, Lorentz transformation can be regarded as a transformation from a rectangular coordinate system to an oblique system.

4+4+4
