2025

MATHEMATICS — HONOURS

Paper : DSCC-6

(Mechanics - I)

Full Marks : 75

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Symbols used in the question have their usual meanings.

Group - A

1. Answer ***any six*** questions : 2×6

(a) State the principle of parallelogram law of forces.

(b) Two forces of magnitudes $3P$, $2P$ respectively have resultant R. If the first force is doubled, the magnitude of the resultant is doubled. Find the angle between the forces.

(c) A particle moves along a straight line, and its distance from a fixed point on the line after t seconds from start is given by $x = a + bt + ct^2$. Prove that it moves with constant acceleration.

(d) What is the principle of conservation of linear momentum?

(e) A particle describes a curve whose polar equation is $r = 6e^{-\theta}$. If the angular velocity is constant, find the cross radial component of acceleration.

(f) The speed v of a point moving along the x axis is given by $v^2 = 16 - x^2$. Prove that the motion is a simple harmonic.

(g) For a rectilinear motion of a particle, if an impulse I changes its velocity from u to v, then show that the change in kinetic energy is $\left(\frac{u+v}{2}\right)I$.

(h) Two bodies of masses m and $4m$ are moving with equal momentum. Find the ratio of their kinetic energy.

(i) Find the law of force, when a particle describes the curve $p^2 = ar$ under a force F to the pole.

(j) State Kepler's laws on planetary motion.

Group - B

2. Answer ***any seven*** questions : 6×7

(a) Forces $\vec{P}$ and $\vec{Q}$, whose resultant is $\vec{R}$, act at a point O. If any transversal cuts the lines of action of the forces $\vec{P}, \vec{Q}, \vec{R}$ at the points L, M, N respectively, then show that $\frac{P}{OL}+\frac{Q}{OM}=\frac{R}{ON}$, where P, Q, R are the magnitudes of the forces $\vec{P}, \vec{Q}, \vec{R}$ respectively.

(b) Three forces of magnitude P, Q, R act along the sides of a triangle formed by the lines $x+y=3$, $2x+y=1$ and $x-y=-1$. Find the equation of the line of action of their resultant.

(c) A particle of mass m is acted on by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance 'a', then show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$.

(d) A particle is moving in a straight line with an acceleration n^2x towards a fixed origin on the line when at a distance x from it and is simultaneously acted on by a periodic force $F\cos pt$ per unit mass, F being a constant. Investigate the motion.

(e) A particle is projected vertically upwards under gravity with a velocity V. Assuming that the resistance of air is kv per unit mass, where v is the velocity of the particle at any instant and k is a constant, show that the particle comes to rest at a height $\frac{V}{K}-\frac{g}{K^2}\log\left(1+\frac{KV}{g}\right)$ above the point of projection, g being the acceleration due to gravity, supposed constant.

(f) One end of an elastic string, whose modulus of elasticity is λ and whose unstretched length is 'a', is fixed to a point on a smooth, horizontal table, and the other end is tied to a particle of mass m which is lying on the table. The particle is pulled to a distance where the extension of the string is 'b' and then let go; show that the time of a complete oscillation is $2\left(\pi+\frac{2a}{b}\right)\sqrt{\frac{am}{\lambda}}$.

(g) A heavy uniform chain of length $2l$, hangs over a small smooth fixed pulley, the length $l+c$ being at one edge and $l-c$ at the other. If the end of the shorter portion be held and then let go, show by the principle of energy that the chain will slip off the pulley in time $\sqrt{\frac{l}{g}}\log\left[\frac{l+(l^2-c^2)^{\frac{1}{2}}}{c}\right], (l>c)$.

(h) A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to height h. Show that the velocity of recoil of the gun is $\left\{\dfrac{2m^2gh}{M(m+M)}\right\}^{\frac{1}{2}}$.

(i) A body of mass M is propelled in a straight line by an engine producing energy at a constant rate P against a resistance kv^2, where v is the velocity and k is a constant. Prove that the space s described from rest is given by $\dfrac{3sk}{M} = -\log\left(1-\dfrac{kv^3}{P}\right)$.

(j) Two perfectly inelastic spheres of masses m_1 and m_2, moving with velocities u_1 and u_2 in the same direction impinge directly. Show that the loss of kinetic energy due to impact is $\dfrac{1}{2}\left(\dfrac{m_1m_2}{m_1+m_2}\right)(u_1-u_2)^2$.

(k) A particle falls from a height h upon a fixed horizontal plane. If e be the coefficient of restitution, show that the whole distance described before the particle has finished rebounding is $\left(\dfrac{1+e^2}{1-e^2}\right)h$ and that the whole time taken is $\sqrt{\dfrac{2h}{g}}\left(\dfrac{1+e}{1-e}\right)$.

Group - C

3. Answer ***any three*** questions : 7×3

(a) Obtain the expressions for components of velocity and acceleration for a particle moving in a plane using polar coordinates.

(b) A particle describes a path with an acceleration $\dfrac{\mu}{y^3}$ which is always parallel to the axis of Y and directed towards the X-axis. If the particle be projected from a point $(0, a)$ with velocity $\dfrac{\sqrt{\mu}}{a}$ parallel to the axis of X, find the path described by the particle.

(c) Establish the differential equation

$$\frac{d^2u}{d\theta^2}+u=\frac{P}{h^2u^2}$$

of the path for the motion of a particle describing a central orbit under an attractive force P per unit mass, the symbols have their usual meanings.

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(d) A particle moving with a central acceleration $\mu(18a^2u^5-8u^3)$ starts from a point at a distance 'a' from the origin in a direction perpendicular to the radius vector and with velocity from infinity. Show that the equation of the path is $r = a\cos 3\theta$.

(e) A planet is describing an ellipse about the Sun as focus. Show that its velocity away from the Sun is greater when the radius vector to the planet is at right angles to the major axis of the path and that it is then $\dfrac{2\pi ae}{T\sqrt{1-e^2}}$, where $2a$ is the major axis, e is the eccentricity and T is the periodic time.

(f) A particle is describing a circle of radius a in such a way that its tangential acceleration is k times the normal acceleration where k is a constant. If the speed of the particle at any point be u, prove that it will return to the same point after a time $\dfrac{a}{ku}\left(1-e^{-2\pi k}\right)$.

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